

**Feasibility Study of Using Centrifugal
Compressor and Expander in a Car
Air Conditioner Working with Carbon
Dioxide as Refrigerant**

Bjørn Fagerli

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For additional information:

Air Conditioning and Refrigeration Center
University of Illinois
Mechanical & Industrial Engineering Dept.
1206 West Green Street
Urbana, IL 61801

(217) 333-3115

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For additional information:

*Air Conditioning & Refrigeration Center
Mechanical & Industrial Engineering Dept.
University of Illinois
1206 West Green Street
Urbana IL 61801*

217 333 3115

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Introduction

A feasibility study has been carried out for applying an electric driven centrifugal compressor with a turboexpander mounted on the same shaft. The shaft is held by magnetic bearings in order to operate with higher speed than today's commercial turbomachinery, but also to obtain an oil free compressor unit. Car air conditioning has been taken as a case study and the intention is to achieve a lighter, smaller, cheaper and more energy efficient compressor unit than the compressors used up today. The refrigerant is carbon dioxide (CO_2), which is considered a potential substitute for R-134a, which is presently used. The main focus of this report is on speed and size, which are assumed to be key issues of applying this compressor type.

Background

There has been a focus on how to make lighter, smaller compressor units in several refrigeration and air-conditioning applications. This is of especially interest in car air conditioning units where there is limited space in the motor compartment. There is also a continuous process of making the cars lighter in order to reduce petrol consumption. Another aspect is to increase the total energy efficiency so the energy consumption of the air conditioner system becomes less energy demanding, which will lead to less petrol consumption.

The size and weight of a compressor can be reduced, keeping the cooling capacity unaltered, on a theoretical level by reducing the swept volume for positive displacement compressor and the suction pocket for rotary compressor but at the same time increasing the shaft speed of the compressor. However if the compressor efficiencies i.e. volumetric and energetic are taken into consideration then the situation is not so simple. Higher speed would imply larger volumetric losses, which requires larger swept volume or suction pocket so there might not be after all so much to gain. When it comes to the energy efficiency, higher speed might reduce the leakage loss i.e. less time available for leakage but on the other hand increase pressure drop losses e.g. in the valves, gas flow passages inside the compressor. Higher speed in traditionally car air conditioner compressors such as piston, scroll and rotary vane might induce larger friction, which is a mechanical loss and therefore energy loss but the friction work is mostly converted into heat. This heat will cause the suction gas to be heated up i.e. the gas density decrease, which would require larger compressor capacity. The friction work, which is not absorbed by the suction gas will be rejected to the ambient environment and therefore be characterized as energy loss.

Other reasons for not speeding up today's compressors are vibration and also wear problems between movable parts. Today, the air conditioner compressor is driven by the engine, which is also limiting factor even if different pulley diameters are used. Gears would of course boost up the compressor speed but this imply extra components and cost, which leads to a more complex unit. Gear components also demand space and would have a certain additional weight.

A centrifugal compressor, also called turbo compressor, is applied in larger air conditioners today, and they can run at quite high speed. However they have never been regarded as suitable for applications with "small" cooling/heating capacity. This might be an obsolete view because there is a continuously development within areas as material, bearing, electric/electronic, manufacturing, assembling, simulation/design etc. so what was not "possible" before can be

“possible” today. A good example of this the development and use of scroll compressor: the concept is about 100 years old but has only be used for 15-20 years since manufacturing obstacles had been overcome.

Advantages and constraints for the proposed idea

What has been the main obstacle for commercial turbo compressor in the low cooling/heating capacity range is the shaft speed, especially for refrigerants with low molecular weight such as ammonia, hydrocarbons etc. This has implied a relatively large impeller diameter to achieve the desired pressure-ratio for a certain “acceptable” shaft speed in a one-stage compressor. Large impeller diameters are problematic because they will cause high material stress and high Mach number at the impeller outlet. The only solution according to traditional turbo compressor practice is to go for several compressor stages, which require more components. This tends to increase both cost and operation complexity.

It is also well known that today’s turbo compressors suffer from a leakage problem when they are scaled down using today’s design practice and at the same time keeping the shaft speed constant.

An interesting question today is if it is possible to use the turbo compressor concept in low cooling capacity range such as in e.g. car air conditioning by applying state of the technology we possess in different engineering areas. The characteristic/parameters should be at least similar or equal but preferable better than the compressor applied today such as:

1. Overall geometric size of the compressor
2. Total weight of the unit
3. Energy efficiency
4. Degree of complexity i.e. to manufacture/assemble
5. Total cost
6. Reliability
7. Cooling capacity control

How can this be achieved? The basic idea is not only to apply an impeller for compressing the refrigerant but also add an expander on the same shaft with an electric motor between those two units. First approach would be to explore if it is possible to achieve desirable pressure ratio in one compression stage.

1. Overall geometric size of the compressor:

Size of the impeller and therefore the overall size is closely connected to the speed. The impeller size can be made small by boosting up the speed. Magnetic radial and axial bearings could be used in order to exceed the shaft speed applied today and not traditional ball and needle bearing.

2. Total weight of the unit:

The total weight can be kept low by keeping the number of components down but also by making the compressor smaller in overall size. The movable parts in this unit would be two impellers and the shaft compared with e.g. seven pistons, seven rods, swash plate etc.

Another advantage would be that this unit would have fewer steel components compared with commercial compressors, which is used today. An assumption is that this unit might utilize aluminum to a greater extent.

An electric motor is included which is not present in today's commercial car air-conditioner. This motor, plus the additional generator capacity will add to the total weight of the unit. However there is no need for a clutch (which has weight) as which is used in today's compressor. However there has been some research (Dieckmann J. and Mallon D. 1992) carried out regarding use of scroll compressor and electric motor for electric cars.

The magnitude of the power output from the electric motor investigated in this study should be less than what is necessary to drive the compressor alone. The reason for this is the expander, which is mounted on the same shaft and will recover expansion work.

3. Energy efficiency:

Two features are suggested to obtain a good energetic efficiency of the unit. The bearing friction would be kept to a minimum since the bearings can be of a magnetic type and not the traditional ball/needle bearings. Higher speed could cause higher viscous friction between moveable part, but on the other hand the parts will be scaled down considerably so the net outcome regarding friction might low. Another feature included to achieve low energy consumption is the expander. This will recover the expansion work, which the refrigerant gas can carry out when it's throttled down from high- to low pressure.

The gas leakage with respect to the compressor impeller would demand special attention in order to minimize the leakage. A positive feature is the fact that the compressor parts will be scaled down compared to the commercial turbo compressor available today. A scale down impeller induces less gas leakage area normal to the leakage flow. It might also be the case that some new technics or device must be invented to keep the leakage at an acceptable level.

4. Degree of complexity i.e. to manufacture/assemble:

This will not be commented at this stage.

5. Total cost:

This will not be commented at this stage.

6. Reliability:

This will not be commented at this stage.

7. Cooling capacity control:

This will not be commented at this stage.

There might be several additional advantages by using this kind of compressor and expander unit such as:

1. This unit does not need lubrication since it is equipped with magnetic bearing. This will first of all save the cost for lubricant in itself but also problems associated with lubricant. Such as lubricant compatibility with materials used in the air conditioner system. There is no chance of compressor failure caused by loss of lubricant. In the end there is no problem regarding recycling with respect to compressor oil.
2. The air conditioner system design might be simpler and easier. There is no need for any oil return system so precautions as high enough gas velocity in the suction gas pipe in order to retain oil back to compressor can be omitted. There is neither any problem if there is a low-pressure receiver in the system regarding miscible oil/refrigerant or that the oil should be heavier than the refrigerant liquid. There is no need for oil-return piping and accessories as valve/orifice so tuning/blockage problems are therefore of no problem.
3. A more energy efficient system might be achieved, measured in higher COP. Lower gas velocity in the suction line can be accepted as a consequence of the absence of lubricant in the system i.e. less pressure drop. Less pressure drop in the suction line implies that the compressor works with a smaller pressure ratio and should therefore achieve higher compressor efficiency. Another aspect with less pressure drop between evaporator outlet and compressor inlet is less expansion of the suction gas, which should give the need for a smaller compressor capacity.

The heat exchangers should also be more efficient when there is no lubricant circulating. The pressure drop should be smaller through the heat exchangers in the absence of lubricant. This would imply less pressure ratio the compressor will work with and normally higher efficiency should be obtained. Another aspect is that there is no lubricant, which might reduce the heat transfer coefficient.

4. This unit does not require a space in the engine compartment any more. It can be placed wherever it is most suitable e.g. in the trunk.
5. It is also suited for electric driven cars since the unit is electrical driven. An idea would be to utilize the electric driven compressor/expander unit both for electric and petrol driven cars and could therefore save production/assembly space and machines.

Impeller, magnetic bearing and motor survey

Magnetic bearing survey

The idea is partly based on high shaft speed, which demands contact free bearing type as gas or magnetic bearing for both the radial and axial bearing. The bearing type, which will be further looked into, is of the magnetic bearing type.

The main advantage of the magnetic bearing is the ability to levitate the compressor shaft with the electric motor rotor and the impellers mounted on before, during and after the shaft rotation.

Gas fills the gap between shaft and stationary-bearing parts so the drag is small compared to contact type bearings. It is claimed that the shaft can run at “any” speed, and the shaft can always be centered. The power needed for such bearings are small compared to the friction work, which arose in contact bearing i.e. ball and needle bearing.

According to (Fulin Gui et al. 1995) magnetic bearings came to use in 1972 in France so the technology is far from a prototype stage. The same source mentions that magnetic bearings have proven to be excellent and reliable at rotational speeds from 10,000 to 800,000 rpm.

In the paper (Ianello V. and Sixsmith H. 1992) there is a theoretical description of one magnetic bearing type and some experimental results of that bearing type, which was design for shaft diameter of 6.4 mm and 500,000 rpm. They divided the test program into three parts; gap sensor test, shock test and rotational test. The intention of the gap sensor test (i.e. clearance between shaft and stationary bearing part) was to see if the type of sensor they used could measure the clearance and then adjust the magnetic force in order to keep constant clearance. The purpose of the shock test was to investigate the stiffness and dampening characteristic in case of sudden movement of the turbomachinery. The rotational test was carried out in order to reveal the stability from start up until maximum speed was achieved and then pull down. The conclusion from the test was that the type of bearing they had developed was suitable for high-speed miniature turbomachinery.

The cost of magnetic bearing is a crucial parameter if this type of bearing is going to be used or not. (Sortore C.K. and Bartosh B.W. 1992) analyzed radial and axial magnetic bearing and concluded that the overall cost of an individual magnetic bearing system produced in a quantity that is equivalent to a conventional bearing system is directly competitive with the cost of that conventional system. They also claim that the magnetic bearing they described in their paper offer significantly advantages over alternative conventional bearing systems i.e. fluid film bearings, ball/needle bearings, air bearings etc. in terms of overall performance, cleanliness, lifetime, flexibility, adaptability, size, investment cost and life cycle cost.

Compressor impeller size survey

What is of interest is to find out different parameters regarding small centrifugal compressors. How small is it possible to make the impellers, what level of energy efficiency can be obtained, what is the mass flow rate would and what kind of pressure ratios are they working with. Such information would give valuable input at this (early) stage. Most compressor impellers mentioned and described in the literature are for centrifugal compressors applied in large chiller units/air conditioners.

Results from a small centrifugal compressor developed for compressing helium is reported in (Peterson T.J. and Fuerst J.D 1986). The compressor test condition:

mass flow rate range:	0.041- 0.080 kg/s
shaft speed range:	50,000 – 80,000 rpm
Pressure ratio interval:	1.2 – 1.5
Adiabatic efficiency range:	37 – 60%

The size of the impeller was not mentioned and the compressor applied gas bearing. The conclusion was that the unit performed as predicted at its design point (60% efficiency, 55 g/s flow at a pressure ratio of 1.4).

A centrifugal compressor is described theoretical in (Sixsmith H. et al. 1988). The compressor is designed for Neon gas and has the follow design parameters:

mass flow rate:	0.001 kg/s
shaft speed:	480,000 rpm
pressure ratio:	1.6
impeller diameter:	1.79 cm

The choice of impeller is of a shrouded type with radial clearance seal surrounding the inlet eye. They estimated a compressor efficiency of 0.4 including an electric motor.

Another paper regarding small centrifugal compressor is (Nobuyoshi S. et. al. 1996). This is also designed to compress helium. It is listed below some compressor data:

mass flow rate:	0.060 kg/s
shaft speed:	80,000 rpm
pressure ratio:	2.8
impeller diameter:	3.30 cm

The compressor gave a small adiabatic efficiency i.e. in the range 40-50% compared to the design efficiency of 70% and 60% outside the design point. They claimed that a filter installed in the inlet pipe caused a reduction of 10-15%. Another source is the different piping size, which would count for another 10% reduction compared to design pipes. The filter and the pipe alteration were carried out by the customer, which also provided the measured data for the efficiency calculations.

An estimate for the smallest impeller diameter based constraints of manufacturability was given by (Jasinski T. et. Al. 1986) to be approximately 5mm.

A common feature regarding the energy efficiencies listed above is that the definition of the applied efficiency is not shown or described in detail.

Expander size survey

The same parameters as for the compressor unit are of interest.

A prototype nitrogen expander was described in (Kunm L.C. and Nenov N.T. 1989). The turbine data is:

shaft speed:	30,000 rpm
impeller diameter:	4.55 cm

This turbine was equipped with ball bearing with mist lubrication, which explains the “low” shaft speed. The turbine efficiency, which was measured, was reported to 87%.

Expansion turbines with much higher shaft speed were reported in (Izumi H. et. al. 1986). Design specifications of the micro turbo expander are shown below:

Turbine number:	1	2	Units
mass flow rate:	0.0028	0.0028	kg/s
shaft speed:	816,000	519,000	rpm
pressure ratio:	2.7	5.1	
impeller diameter:	0.6	?	cm

The authors of the paper mentioned that they were working on two different bearings i.e. tilting pad bearing and bush type bearing, which enable them to run the turbine at such high speeds. The efficiencies for these two turbines are in the 40-50% range.

A turbo expander was also developed by (Sixsmith H. et al. 1988), which was referred earlier regarding a turbo compressor. Some design parameters for their turbine are:

shaft speed:	570,000 rpm
impeller diameter:	0.318 cm

Their goal for overall turbine efficiency was 50%.

An expander in an air conditioner system must work with a wet outlet i.e. that the refrigerant is in the two-phase condition. A centrifugal turbine working under such condition should not cause any problem according to (Sixsmith H. et. al. 1990), which have designed and tested such turbine under the wet outlet condition. The design specifications of the wet turboexpander working with helium are:

mass flow rate:	0.050 kg/s
shaft speed:	384,000 rpm
pressure ratio:	8.6
impeller diameter:	0.476 cm

The impeller diameter was the same as for the shaft, which they mentioned to offer significant advantages in fabrication, assembly and performance. This design avoids the need for precise axial location of the shaft and permits the shaft to be fabricated from a single piece of metal. The isentropic efficiency was measured to about 50%, which was less than anticipated but where explained with some debris, which was clogged in the turbine nozzle. The experimental results with the wet turboexpander were so good that the authors said it could replace J-T (Joule Thomson) stage and wet engine stages in laboratory sized helium liquefiers.

Electric motor survey

An interesting question: what is the upper limit for the speed of electrical motors? The compressor impeller survey chapter reveals that a speed of 480,000 rpm (without the use of gear)

can be obtained (Sixsmith H. et al. 1988). The motor they used was low voltage ac three phase induction motor with solid rotor. The laminations for the stator were fabricated from a low loss magnetic alloy. They didn't give any dimensions e.g. for stator diameter but included a cut away drawing of the compressor with the electric motor. An estimate for the outer stator diameter is 2.5 cm scaled from the drawing and based on the stated compressor impeller diameter of 1.79 cm.

A literature search in the Inspec and Compendex data did not provide any hits regarding electric motors operating above 200,000 rpm.

Design operating condition for compressor and the expander unit

One approach to establish the design operating condition when using carbon dioxide (R-744) as a refrigerant is to first look at a frequently used design operating condition for a R-134a system i.e. ambient air temperatures. Then decide the R-744 pressures based these temperatures and some assumption of temperature differences. The size of a R-134a compressor i.e. swept volume is determined from a desired cooling capacity at idle condition i.e. when the car is parked and the engine is running at lowest rpm. In the following calculation a cooling capacity of 3000W is applied, which is typical for a middle sized car.

The aim was to obtain more or less equal cooling capacity for the R-744 as for the R-134a compressor at equal temperature condition. A carbon dioxide system with low-pressure receiver and internal heat exchanger was applied in the calculations.

The following constraints/assumptions were applied:

1. A maximum discharge pressure of 130 bar and 150°C for the discharge gas temperature. The pressure constraint was based on a desire to keep the wall thickness of heat exchanger and compressor house on a moderate level. The gas temperature constraint was based on potential problem with gasket/o-ring material, hoses etc.
2. It was assumed that the evaporating temperature for the R-744 could be set 5°C higher than the R-134a system (average temperature difference through the evaporators). This might be a bit optimistic according (Yin J. et.al. 1998), which has measured a temperature difference of 2.2°C at the evaporator outlets. The evaporator in a R-744 system has a larger total heat transfer surface on the air-side compared to a R-134a evaporator even if the core volume of both evaporators are equal. The reason for this is much smaller volume or cross flow area on the refrigerant side for the R-744 compared to the R-134a evaporator. Larger heat transfer surface area on the air-side implies higher evaporating temperature.
3. Another assumption for the R-744 system was 5°C between refrigerant leaving the gas cooler and the ambient air temperature entering the gas cooler. This assumption of 5°C might be a bit pessimistic since (McEnaney R. P. et.al. 1999) has measured lower temperature difference at different operating conditions.
4. The last assumption was a suction gas superheating of 15°C before the gas enters the compressor.

Table 1

Parameter	*Idle condition		Units
	R-134a	R-744	
Evaporating temperature	9.7	14.7	°C
Evaporating pressure	4.1	50.4	bar
Gas temperature compressor inlet	18	29.7	°C
High-side pressure	24.7	145	bar
Condensing temperature	77.0	-	°C
Temp. condenser/gas cooler outlet	71.2	66	°C
Pressure difference	20.6		bar
Pressure ratio	6.0		-

Ambient air temperature is set to 43.3°C

**Air temperature into the gas cooler/condenser 61.1°C*

Theoretical optimum COP the R-744 system at the idle condition was 145 bar including the expander i.e. recovering of the expansion work and additional cooling capacity. This is higher than the discharge pressure constraint. The discharge gas temperature was 118°C for the idle condition.

Isentropic compression head and ideal mass flow rate is necessary to establish in order to calculate different impeller parameters and shaft speeds, which will later be adjusted with assumption of total volumetric loss and energy efficiency.

The table below shows the influence the suction gas superheating in the internal heat exchanger on the isentropic compression head, isentropic expansion work and ideal gas mass flow rate at idle condition when the cooling capacity (3000W) and the discharge pressure 130 bar are set constant.

Table 2

Parameter	Idle condition (R-744)					Units
Suction gas superheating:	5	10	15	20	25	°C
Isentropic compression head:	38.5	20.1	19.3	18.2	17.2	kJ/kg
Isentropic expansion work:	22.1	20.1	19.3	18.2	17.2	kJ/kg
COP	6.1	5.8	5.6	5.5	5.5	-
Ideal mass flow rate:	0.049	0.042	0.038	0.035	0.033	kg/s
Discharge gas temperature:	95	102	109	115	121	°C

NB! The cooling capacity is 3000W

It can be seen from the table above that the theoretical COP maximum is achieved with minimum superheating of the suction gas. Another series of calculation are carried out with 5°C suction gas superheating.

Table 3

Parameter	Idle condition (R-744)					Units
Discharge pressure:	110	120	130	140	150	bar
Isentropic compression head:	30.9	34.8	38.5	42.0	45.3	kJ/kg
Isentropic expansion work:	22.6	22.4	22.0	21.7	21.7	kJ/kg
COP	4.1	5.5	6.1	6.3	6.3	-
Ideal mass flow rate:	0.106	0.066	0.049	0.040	0.035	kg/s
Discharge gas temperature:	81	88	95	101	107	°C

NB! The cooling capacity is 3000W

Theoretical COP was also for 5°C suction gas superheating obtained above the discharge pressure constraint of 130 bar. Table 4 shows the operating condition, which will be used to estimate the compressor impeller parameters

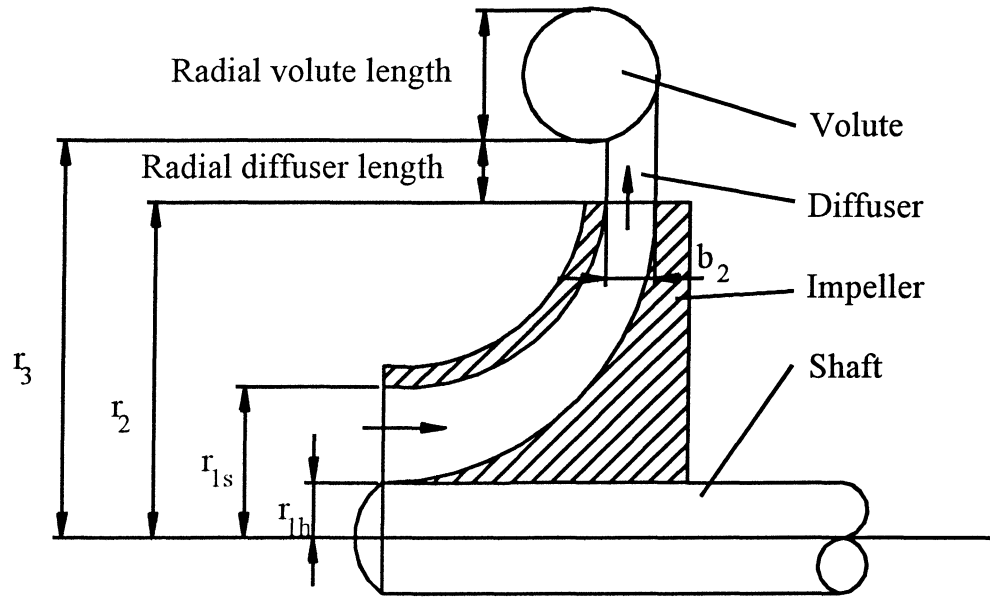
Table 4

Parameter	*Idle condition	Units
Evaporating temperature	14.7	°C
Evaporating pressure	50.4	bar
Gas temperature compressor inlet	19.7	°C
High-side pressure	130	bar
Temp. condenser/gas cooler outlet	66	°C
Pressure difference	79.6	bar
Pressure ratio	2.6	-

Compressor impeller calculations

In this preliminary study it is of interest to obtain estimates for certain parameters such as shaft speed, impeller diameter etc. A simple procedure is listed in Appendix A in order to calculate parameters of interest based operating condition i.e. gas pressures, temperature and mass flow rate. In addition it is necessary to assume impeller efficiency, total energy efficiency and flow angles at the inlet and outlet of the impeller.

The figure below shows a simplified cut away drawing of an impeller. The figure shows the parameters, which are of interest at this stage and the simple procedure listed in Appendix A will be applied to quantify these parameters (except r_3) in addition to the shaft speed.



where: the character 1 and 2 mean respectively inlet and outlet to the impeller
 r_{1h} – hub radius
 r_{1s} – shroud radius
 r_2 – impeller radius
 b_2 – impeller height
 r_3 – the total radius of impeller and diffuser

There are also some parameters, which must be decided/assumed in addition to the operating condition i.e. pressures, temperature and mass flow rate.

Parameters which must be specified:

- μ - slip factor
- η_i – impeller total to total efficiency
- η_s – stage total to total efficiency
- β_{1s} – inlet shroud relative flow angle
- β_{b2} – impeller discharge relative flow angle
- α_2 – impeller discharge absolute flow angle
- $v(r_{1h}/r_{1s})$ - inducer hub to shroud radius ratio
- r_{1s}/r_2 – impeller inducer radius to discharge tip radius

The slip factor (μ) was set to 0.8.

The impeller stage efficiency (η_i) was set to 0.85

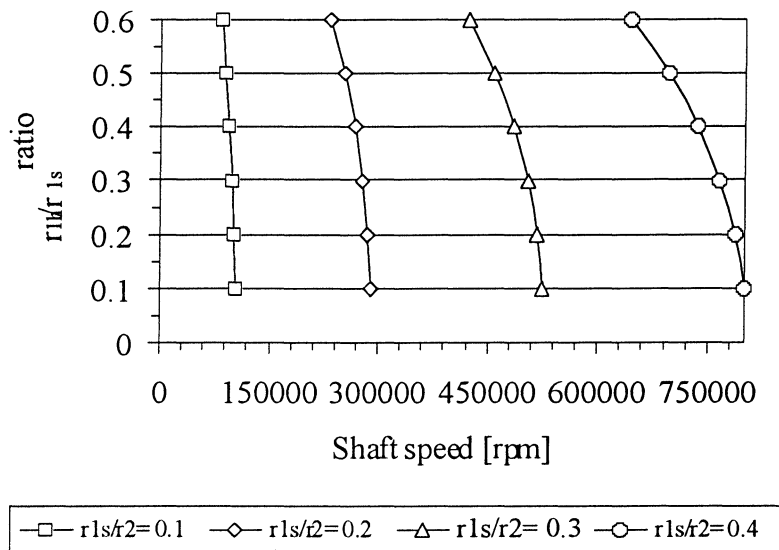
The stage total efficiency (η_s) was set to 0.6

The inlet shroud relative flow angle (β_{1s}) was set to -56°

The impeller discharge relative flow angle (β_{b2}) was set to 0°

The impeller discharge absolute flow angle (α_2) was set to 85°

The first calculations are shown in the figure below. It has been decided to plot the two ratios i.e. r_{1h}/r_{1s} and r_{1s}/r_2 versus shaft speed.



It is desired to choose as “high” shaft speed as possible since size of the impeller is closely connected to the shaft speed. A shaft speed of 350000 rpm will be applied as design speed based on the reported speed of 480000 rpm. Then it could be possible to exceed this the design speed if required.

The ratio r_{1s}/r_2 should be low in order to make the compressor compact and this is obtained with a shaft speed of 350000 rpm. A high value of the r_{1h}/r_{1s} would give a small flow area at the inducer i.e. high Mach number. So from energy point of i.e. low gas velocity point of view this should be low.

Initial ratio values and speed.

Shaft speed: 350000 rpm
 r_{1s}/r_2 ratio: 0.25
 r_{1h}/r_{1s} : 0.5

This choice together with the design condition and mass flow gives:

Mach number at the inducer: 0.22
Mach number at the impeller outlet: 0.94
Hub radius (r_{1h}): 1.2 mm
Shroud radius (r_{1s}): 2.4 mm
Impeller radius (r_2): 9.6 mm
Impeller height (b_2): 0.3 mm

As a rule of thumb, the outer diameter of the diffuser is in the range of 1.5 to 2.0 times the impeller diameter. This gives:

The total radius of impeller and diffuser (r_3): 14.4 – 19.2 mm

or

Radial diffuser length: 4.8 – 9.6 mm (impeller diameter minus total radius)

It must be emphasized that the calculations have been carried out with any concern about friction or other losses. Neither are the number of impeller blades and blade thickness taken into consideration. The flow inlet area might be adjusted when number of blades and the blade thickness are included but not so much since the Mach number is very low. The impeller height should be adjusted when friction and blades are taken into consideration. This means that the height must be increased in order to keep the flow area normal the impeller unaltered.

It is a normal practice to at least start the diffuser width with the same magnitude as the impeller height. Since this is small (even after a correction for friction and blades) then the rule of thumb, which is applied to calculate the outer diameter i.e. 1.5 – 2 times the impeller diameter might not be valid. So this factor might be lower than 1.5 – 2 times.

The largest diameter of the volute can be estimated by assuming a low velocity at the volute outlet e.g. 5 m/s. This gives an approximately volute outlet diameter of 10 mm. The volute can be put atop of the diffuser or beside as shown in the simplified sketch in the beginning of this chapter. The last option will induce a more compact compressor/expander unit.

A compressor house wall thickness of 10 mm (aluminum material) should be sufficient according to dimensioning tube wall rules based on the diameter and pressures, which have been estimated.

An estimate for the total outer radius is now possible to achieve when the impeller, diffuser, volute and the compressor house thickness are taken into consideration:

	Maximum	Minimum	Units
Impeller radius	9.6	9.6	mm
Radial diffuser length	9.6	4.8	mm
Radial volute length	10	5	mm
Compressor house thickness	10	10	mm
Total outer radius	39.2	29.4	mm

The outer radius for a R-134a variable displacement compressor is about 55 mm so it is possible to reduce the outer radius in the range of 29 – 46% compared to the state of the art compressor.

Nomenclature

a	speed of sound
b	impeller height
C	absolute velocity
M	Mach number
n	specific speed
N	speed of rotation
P	pressure
PR	pressure ratio
r	radius
T	temperature
W	relative velocity
\dot{W}	power
U	tangential blade speed
α	absolute flow angle
β	relative flow angle
ϕ	flow coefficient
γ	specific heat ratio
η	efficiency
λ	work input factor
μ	slip factor
v	diameter ratio
ρ	density
ψ	blade loading coefficient

Subscript:

0	total state
1	inlet
2	outlet
h	hub
I	impeller
s	shroud, stage

Appendix A

1.
$$\lambda = \frac{\mu}{1 - \frac{\tan \beta_{B2}}{\tan \alpha_2}}$$
2.
$$\frac{U_2}{a_{01}} = \frac{PR^{\frac{\gamma-1}{\gamma}} - 1}{\eta_s \lambda (\gamma - 1)}^{\frac{1}{2}}$$
3.
$$\frac{C_{\theta 2}}{a_{01}} = \frac{U_2}{a_{01}} \lambda$$
4.
$$\frac{T_{02}}{T_{01}} = 1 + (\gamma - 1) \frac{U_2 C_{\theta 2}}{a_{01} a_{01}}$$
5.
$$\frac{P_{02}}{P_{01}} = \eta_I \frac{-T_{02}}{T_{01}} - 1 \sqrt[\frac{\gamma}{\gamma-1}]{+1}$$
6.
$$\frac{C_2}{a_{01}} = \frac{C_{\theta 2}}{a_{01}} \frac{1}{\sin \alpha_2}$$
7.
$$\frac{C_2}{a_{02}} = \frac{C_2}{a_{01}} \frac{-T_{01}}{T_{02}} \sqrt[\frac{1}{2}]{+1}$$
8.
$$\frac{T_2}{T_{02}} = 1 - \frac{\gamma - 1}{2} \frac{C_2}{a_{02}} \sqrt[\frac{1}{2}]{+1}$$
9.
$$M_2 = \frac{C_2}{a_{02}} \frac{-T_{02}}{T_2} \sqrt[\frac{1}{2}]{+1}$$
10.
$$\frac{U_2}{a_{02}} = \frac{U_2}{a_{01}} \frac{-T_{01}}{T_{02}} \sqrt[\frac{1}{2}]{+1}$$
11.
$$\frac{U_2}{a_2} = \frac{U_2}{a_{02}} \frac{-T_{02}}{T_2} \sqrt[\frac{1}{2}]{+1}$$
12.
$$\frac{C_{\theta 2}}{a_2} = \frac{C_{\theta 2}}{a_{01}} \frac{-T_{02}}{T_2} \frac{T_{01}}{T_{02}} \sqrt[\frac{1}{2}]{+1}$$
13.
$$\frac{W_{\theta 2}}{a_2} = \frac{C_{\theta 2}}{a_2} - \frac{U_2}{a_2}$$

$$14. \quad \frac{C_{m2}}{a_2} = M_2 \cos \alpha_2$$

$$15. \quad M'_2 = \frac{-W_{\theta 2}}{a_2} \sqrt{\frac{2}{\gamma-1}} + \frac{-C_{m2}}{a_2} \sqrt{\frac{2}{\gamma-1}}$$

$$16. \quad \beta_2 = \tan^{-1} \frac{\frac{-W_{\theta 2}}{a_2} \sqrt{\frac{2}{\gamma-1}}}{\frac{-C_{m2}}{a_2} \sqrt{\frac{2}{\gamma-1}}}$$

$$17. \quad \frac{P_{02}}{P_2} = \frac{-T_{02}}{T_2} \sqrt{\frac{\gamma}{\gamma-1}}$$

$$18. \quad \frac{P_2}{P_{01}} = \frac{P_{02}}{P_{01}} \frac{P_2}{P_{02}}$$

$$19. \quad \frac{\rho_2}{\rho_{01}} = \frac{P_2}{P_{01}} \frac{T_{02}}{T_2} \frac{T_{01}}{T_{02}}$$

$$20. \quad \frac{C_{m2}}{a_{01}} = \frac{C_{\theta 2}}{a_{01}} \frac{1}{\tan \alpha_2}$$

$$21. \quad \frac{U_1}{a_{01}} = \frac{U_2}{a_{01}} \frac{r_{1s}}{r_2}$$

$$22. \quad \frac{W_{\theta 1}}{a_{01}} = -\frac{U_1}{a_{01}}$$

$$23. \quad \frac{C_1}{a_{01}} = \frac{W_{\theta 1}}{a_{01}} \frac{1}{\tan \alpha_{1s}}$$

$$24. \quad M_1 = \frac{C_1}{a_{01}} \left[1 - \frac{\gamma-1}{2} \frac{-C_1}{a_{01}} \sqrt{\frac{2}{\gamma-1}} \right]^{-\frac{1}{2}}$$

$$25. \quad M'_1 = \frac{M_1}{\cos \beta_{1s}}$$

$$26. \quad M_R = \frac{M'_1}{M'_2}$$

$$27. \quad \frac{W_{1s}}{a_{01}} = \frac{-W_{\theta 1}}{a_{01}} \sqrt{\frac{2}{\gamma-1}} + \frac{-C_1}{a_{01}} \sqrt{\frac{2}{\gamma-1}}$$

$$28. \quad \frac{W_2}{a_{01}} = M_2' \frac{T_{02}}{T_{01}} \frac{T_2}{T_{02}} \sqrt{\frac{1}{2}}$$

$$29. \quad W_R = \frac{\frac{W_{1s}}{a_{01}}}{\frac{W_2}{a_{01}}}$$

$$30. \quad \frac{\rho_1}{\rho_{01}} = \frac{1}{1 + \frac{\gamma-1}{2} M_1^2} \sqrt{\frac{1}{\gamma-1}}$$

$$31. \quad \psi = 2\lambda\eta_s$$

$$32. \quad \phi = \frac{\rho_1}{\rho_{01}} \frac{r_{1s}}{r_2} \sqrt{\frac{1}{2}} (1-v^2) \frac{C_1}{a_{01}} \frac{a_{01}}{U_2}$$

$$33. \quad \theta = \phi \frac{U_2}{a_{01}}$$

$$34. \quad \frac{b_2}{r_2} = \frac{\theta}{\frac{1}{2} \frac{\rho_2}{\rho_{01}} \frac{C_{m2}}{a_{01}} \sqrt{\frac{1}{2}}}$$

$$35. \quad N_s = \frac{(\pi\phi)^{\frac{1}{2}}}{\frac{\psi}{2} \sqrt{\frac{1}{4}}}$$

$$36. \quad n_s = \frac{\pi \frac{r_{1s}}{r_2} \sqrt{\frac{1}{2}} (1-v^2) \frac{C_1}{a_{01}} \frac{a_{01}}{U_2}^{\frac{1}{2}}}{(\lambda\eta_s)^{\frac{3}{4}}}$$

$$37. \quad \dot{W}_{ND} = \psi \theta \frac{U_2^2}{a_{01}} \sqrt{\frac{1}{2}}$$

$$38. \quad \rho_{01} = \frac{P_{01}}{RT_{01}}$$

$$39. \quad a_{01} = (\gamma RT_{01})^{\frac{1}{2}}$$

$$40. \quad A_2 = \frac{\dot{m}}{\theta \rho_{01} a_{01}}$$

$$41. \quad r_2 = \frac{-A_2}{\pi} \sqrt{\frac{1}{2}}$$

$$42. \quad b_2 = \frac{b_2}{r_2} r_2$$

$$43. \quad r_{1s} = \frac{r_{1s}}{r_2} r_2$$

$$44. \quad r_{1h} = v r_{1s}$$

Reference

- Dieckmann J. and Mallon D. 1992
Variable speed compressor, HCF-134a based air conditioning system for electric vehicles, SAE Technical paper series #920444, pp. 606-610
- Fulin G. et. al. 1995
Design and experimental study of high-speed low-flow rate centrifugal compressor, IECEC paper No. CT39, ASME, pp. 35-41
- Iannello V. and Sixsmith H. 1992
Magnetic bearings for cryogenic turbomachines, Advances in Cryogenic Engineering, pp. 809-816
- Izumi H. et. al. 1986
Development of small size Claude cycle helium refrigerator with micro turbo-expander, Advances in Cryogenic Engineering, pp. 811-818
- Jasinski T. et. Al. 1986
A generic pump/compressor design for circulation of cryogenic fluids, Advances in Cryogenic Engineering, pp. 991-997
- Kunm, L.C. and Nenov N.T. 1989
Liquid nitrogen expansion turbine, ASME paper in Cryogenic processes and equipment, pp. 79-83
- Nobuyoshi S. et. al. 1996
Design, manufacture and consideration for test result of cold compressor for TEVATRON lower temperature upgrade, ICRC16/ICMC proceedings, pp. 207-210

- Peterson T.J. and Fuerst J.D. 1986
Tests of cold helium compressors at FERMILAB, Advances in Cryogenic Engineering, 655-662
- McEnaney R.P., Park Y.C., Yin J.M. and Hrnjak P.S. 1999
Performance of the prototype of a transcritical R-744 mobile ac system, SAE paper 1999-01-0872
- Sixsmith H. et. al. 1990
A miniature wet turboexpander, Advances in Cryogenic Engineering, pp. 989-995
- Sixsmith H. et. al. 1990
Small turbo - Brayton cryocoolers, Advances in Cryogenic Engineering, pp. 827-836
- Sortore C.K. and Bartosh B.W. 1992
Compact and cost effective active magnetic bearing system design for high speed rotors, Journal of the society of tribologists and lubrication engineers, vol. 49, pp. 555-560
- Yin J., Park Y.C., Boewe D., McEnaney R., Beaver A., Bullard C.W. and Hrnjak P.S. 1998
Experimental and model comparison of transcritical CO₂ versus R-134a and R-410 system performance, IIR conference, Proceeding "Natural Working Fluid '98", Oslo, Norway, pp.331 - 340